In this report, we implemented a simple linear regression of the Dow Jones Industrial Average (“DJIA”) index over the S&P500 (“S&P”) using daily and annual log returns.

Our key finding is that the return characteristics of the DJIA does not exhibit statistically significant average excess returns as compared to the S&P.

Another finding is that using different time intervals (daily vs annual log returns) to preform the regression leads to significant differences in the kurtosis measure of the distribution of the error terms in the regression. Investigating further, the reason for this difference is found to be due to the bias introduced into the annual data series when the last day of the year is used as a benchmark. We present further comparative illustrations to show this.

**1. Task 3 – Regression using daily log returns**

**1.1 Estimate of key statistics \hat{a} \hat{b} and \hat{\sigma\_u}**

\hat{a} 0.00005

\hat{b} 0.94311

\hat{\sigma\_u} 0.00288

The regression results in an intercept of 0.005% and a slope of 0.9431.

The positive intercept indicates that the DJIA has a small positive daily excess returns on average as compared to the S&P.

In this context, the slope of 0.9431 is interpreted as Beta – the sensitivity of DJIA’s returns to the S&P’s returns. Assuming investors are risk-averse and that the CAPM model fully describes the market, a lower Beta is preferred for the same level of return because a lower Beta asset will have less variance in its returns.

Combining these two measures together, it suggests that the DJIA may have a superior risk-adjusted return as compared to the S&P.

**1.2 t-test for Null Hypothesis a=b=0 at 5% significance**

The t-test statistic for \hat{a} is 1.48636

The t-test statistic for \hat{b} is 339.97

Upper and lower critical values: ± 1.96024

We conduct the t-test to deduce if our estimates of \hat{a} and \hat{b} are significant at the 5% level.

Our sample size is 8500 observations, leading to 8498 degrees of freedom

The test statistic for \hat{a} falls within the critical values, and thus we cannot reject the null hypothesis that \hat{a}=0. The indication of DJIA having higher daily returns than the S&P is therefore not significant at the 5% level.

However, the test statistic for \hat{b} falls outside the critical values, and thus we reject the null hypothesis that \hat{b}=0 and conclude that there is a linear relationship between DJIA and S&P at the 5% significance level.

**1.3 R^2 and Adjusted R^2 values**

R^2 93.15120 %

Adjusted R^2 93.15039 %

The R^2 value is very high, further supporting our previous point that there exists a linear relationship between the daily returns of the DJIA and the S&P.

**1.4 Jarque-Bera test statistic for the residuals**

JB statistic (\hat{u}): 25434.27

5% Critical Chi-Square Value with 2 Degrees of Freedom: 5.99146

The JB test statistic exceeds the critical value at the 5% significance by a huge margin, strongly indicating that the residuals are not normally distributed. This is due to regression outliers that were a result of extreme market conditions, for example the huge one-day drop on 19-Oct-1987.

**1.5 Additional test - H\_0: \hat{b}=1**

We conduct an additional test of \hat{b}=1 at the 5% significance level to determine if our \hat{b} value of 0.94311 is statistically different from 1. We do this to gauge how well the returns of the DJIA mimic the returns of the market portfolio S&P.

The t-test statistic for \hat{b}=1 is -20.50787

Upper and lower critical values: ± 1.96024

We reject the null hypothesis that \hat{b}=1, and conclude that the daily log returns of the DJIA do not perfectly mimic the returns of the S&P. Therefore, using the DJIA as a benchmark for performance evaluation may lead to different results as compared to when using the S&P as the benchmark.

**2. Task 4 – Regression using annual log returns**

**2.1 Estimate of key statistics \hat{a} \hat{b} and \hat{\sigma\_u}**

\hat{a} 0.01978

\hat{b} 0.84254

\hat{\sigma\_u} 0.03797

The regression results in an intercept of 1.978% and a slope of 0.8425.

The positive intercept indicates that the DJIA has a small positive daily excess returns on average as compared to the S&P.

The slope of 0.8425 indicates that the DJIA is slightly less volatile than the S&P within the CAPM framework.

Combining these two measures together, it suggests that the DJIA has a superior risk-adjusted return as compared to the S&P.

**2.2 t-test for Null Hypothesis a=b=0 at 5% significance**

The t-test statistic for \hat{a} is 2.64989

The t-test statistic for \hat{b} is 20.4364

Upper and lower critical values: ± 2.04227

We conduct the t-test to deduce if our estimates of \hat{a} and \hat{b} are significant at the 5% level.

Our sample size is 32 observations, leading to 30 degrees of freedom

The test statistic for \hat{a} falls outside the critical values, and thus we reject the null hypothesis that \hat{a}=0. The t-test at 5% significance concludes that the DJIA has higher annual returns as compared to the S&P.

The test statistic for \hat{b} also falls outside the critical values, and thus we reject the null hypothesis that \hat{b}=0 and conclude that there is a linear relationship between the annual returns of the DJIA and the S&P at the 5% significance level.

**2.3 R^2 and Adjusted R^2 values**

R^2 0.932983

Adjusted R^2 0.930749

The R^2 value is very high, further supporting our previous point that there exists a linear relationship between the annual returns of the DJIA and the S&P.

**2.4 Jarque-Bera test statistic for the residuals**

JB statistic (\hat{u}): 1.04637

5% Critical Chi-Square Value with 2 Degrees of Freedom: 5.99146

The JB test statistic falls within the critical value at the 5% significance level, indicating that the regression residuals are normally distributed.

**2.5 Additional test - H\_0: \hat{b}=1**

We conduct an additional test of \hat{b}=1 at the 5% significance level to determine if our \hat{b} value of 0.84254 is statistically different from 1. We do this to gauge how well the returns of the DJIA mimic the returns of the market portfolio S&P.

The t-test statistic for \hat{b}=1 is -3.81917

Upper and lower critical values: ± 1.96024

We reject the null hypothesis that \hat{b}=1, and conclude that the annual log returns of the DJIA do not perfectly mimic the returns of the S&P. Therefore, using the DJIA as a benchmark for performance evaluation may lead to different results as compared to when using the S&P as the benchmark.

**3. Additional Commentary**

The vastly differing results from the Jarque-Bera test performed on the residuals of the daily versus the annual returns led us to investigate further as to the possible reason why this may be so.

The density histograms show the stark difference in the distribution of the residuals. The density histogram for the annual returns is relatively flat, while the one for the daily returns show a sharp peak. The impact of outliers is therefore much more pronounced for the daily returns.

There are also a greater number of outliers in the daily return residuals. We noticed that annual returns used the closing value of the last business day of the year. This has the effect of dampening observed volatility, as price swings occurring during the course of the year are not captured. This, coupled with the observation that December tends to be one of the calmer months, lead a downward bias in the volatility measure, which feeds into the regression.